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ICME 13
Hamburg 2016

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13th International Congress on
Mathematical Education 2016
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Hamburg, 24/Jul/2016

CERTIFICATE

Confirmation of participation in the 13th International Congress on Mathematical Education 2016 and receipt of payment of the congress fee

Herewith we confirm that **Dr. Tatag Yuli Eko Siswono** has participated actively in 13th International Congress on Mathematical Education 2016, held from 24th to 31st July 2016 in Hamburg Germany.

Dr. Tatag Yuli Eko Siswono is author/co-author of the following accepted contribution(s):

HOW MATHEMATICALLY LITERATE ARE SECONDARY TEACHERS IN PERFORMING CONTEXT-BASED PROBLEM SOLVING TASK?:A CASE OF INDONESIA
Author(s): Siswono, Tatag Yuli Eko
Presenting Author: Siswono, Tatag Yuli Eko
Submission Type / TSG Number: TSG 23 - Mathematical literacy

Furthermore, we confirm the receipt of the congress fee of EUR 390.00.

With best regards,

Gabriele Kaiser

Prof. Dr. Gabriele Kaiser

13th International Congress on Mathematical Education
Hamburg, 24-31 July 2016

HOW MATHEMATICALLYLITERATE ARE SECONDARY TEACHERS IN PERFORMING CONTEXT-BASED PROBLEM SOLVING TASK? : A CASE OF INDONESIA

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This study aims to look into Indonesian secondary teachers' performance on context-based problem solving task based on perspective of PISA view on mathematical literacy.Results of studying teachers' incorrect answers on PISA like task point out thattransformation error appeared as the most frequently found in teacher's performance. This is reinforced by data ofless activation ofa set of fundamental mathematical capabilities mentioned in PISA framework 2012 appeared on these answers.Clarifying with questionnaire result, teachers admitted to most frequently failin determining a precise mathematical model or strategies when carrying out context-based problem solving. The consequences of these results regarding both teacher education on mathematical literacy and further researches are then discussed related to reflection on teachers' own formal conception of what it means to be mathematically literate.

Keywords: mathematical literacy, context-based task, problem solving, mathematics teacher

INTRODUCTION

It is highlighted that success story of students' mathematical literacy performance for some countries as reported by PISA(Programme for International Student Assessment)is inevitably influenced by the success story of developing teachers' quality on this issue. In particular, Stacey (2011) argued that countries such as Finland, Singapore, and Japan each of which scored highly in PISA surveyconsistently from the beginning survey in 2000, are often attributed to theirgreat emphasis on concerning teacher's quality improvement.Considering this emphasis, Brown and Schafer (2006) has broadly structured their teacher education program on developing teachers' own, more formal, conception of what it means to be mathematically literate.

Being mathematically literate, students as well as teachers should be able to identify and use relevant mathematics to a variety of context (OECD,2013a). In performing this competence, they need to complete mathematical process as mentioned by PISA framework (OECD,2013a) well, i.e from formulating problem mathematically, employing mathematical procedures and facts, to interpreting the mathematical results back to initial problem. Moreover, a set of mathematical competencies, namely Fundamental Mathematical Capabilities (FMC) developed by PISA team are also needed to be activated into each of the mathematical processes with a variety of degrees of complexity.

In Indonesia, mathematical literacy has been becoming important issue, particularly, because of results on students' insignificant improvement in PISA survey since 2003. A great number of researches regarding students' performance on this issue, actually, has been conducted with a variety of purposes such as developing context-based task (see for example, Dewantara, Zulkardi, & Darmawijoyo, 2015;Kamaliyah, Zulkardi, & Darmawijoyo, 2014)and investigating students' difficulties (Kohar and Zulkardi, 2015,Wijaya, van den Heuvel-Panhuizen, Doorman, and Robitzsch, 2014). However, research on how Indonesian teachers bring out idea of mathematical literacy as understanding context-based problem solving task are limited to be found.This issue can be analyzedfrom teachers' mathematical literacy knowledge as mentioned by Bansilal, Webb,& James (2015), i.e.content knowledge, pedagogic content knowledge, and reflective practice. As preliminary study, we investigated their mathematical literacy content knowledge through theirperformance in solving context-based task as well as their difficulties regarding this issue to get them reach common sense conception of being mathematicallyliterate.Thus, the following research questions were addressed: (1) How do secondary teachers perform context-based problem solving task based on PISA's mathematical literacy process? (2) What are secondary teachers' difficulties in(a) viewing context-based problem solving task, and (b) performing context-based problem solving task regarding PISA's fundamental mathematical capabilities?

THEORITICAL FRAMEWORK

PISA view on Mathematical Literacy

Mathematicalliteracy is formally defined by OECD (2013a) as an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. In each of the processes: 'formulate', 'employ', and 'interpret', FMC are activated successively and simultaneously in varying degrees depends on the related mathematics topics to obtain a solution (OECD,2013a). FMC consist of (1) communication, (2) reasoning and argumentation, (3) mathematising, (4) representation, (5) devising problem solving strategies, (6) using symbolic, formal and technical language and operations.

Context-based Problem Solving Task (CbPST)

Problem solving task refers to task which is challenging, offers an opportunity to extend and/or generalize, means to the goal of transfer of problem solving skills to other situations, consider the importance of assessing process of solving the task, and accentuate Polya's problem solving model to guide the development of the problem solving tasks (Dindyal, Tay, Quek, Leong, Toh, Toho, Ho, 2013). Meanwhile, context-based task refers to task situated in realworld settings which provide elements or information that need to be organized and modeled mathematically (Wijaya, et al, 2014). A 'context problem' should be authentic for learners and should require them to think within the context of the problem. Hence, PISA has used almost no zero-order context items (Salgado& Stacey, 2014), i.e. the items whose context is a mere camouflage, generally involves just mathematical terms, shapes, data and the translation of textually packaged mathematical problems. Rather, PISA used first order and second order use of context, each of whose context is relevant and needed for solving the problem and the knowledge of context is needed to "mathematize" the problem in order to solve it respectively(OECD, 2009). Accordingly, context-based problem solving task (CbPST) refers to task which accommodates real world authentic context as source of problemof the task needed to be solved by providing a precise mathematical model, carrying out some mathematical procedures/facts to get mathematical results and then giving appropriate interpretation to the initial problem.

Solving context-based task needs specific process since the type of its tasksituation which is presented in real world problem. In processing the first step of Polya's problem solving, understanding problem, for instance, one need to state the real world problem in his/her own words, pinpoint exactly what is being asked, identify the unknowns, and identify any information that is irrelevant to the problem. A prominent process has been proposed and developed by OECD (2013) which regards to solve context-based mathematics task: formulate, employ, and interpret. Theseprocesses seems correspond with other type of steps of solving task, for instance, Polya's problem solving (1973),andNewman Analysis (Clements, 1980). The comparison among those three processes is shown below.

Polya's Problem Solving Process(Polya, 1973)	Newman Analysis (Clements, 1980)	PISA's Mathematical Literacy (OECD, 2013a)
Understandingtheproblem: identifytheunknown, data, and condition related to the information given in the problem	*Reading*: recognize of words and symbols *Comprehension:* understand the meaning of a problem	*Formulate:* recognise and identify opportunities to use mathematics and then provide mathematical structure to a problem presented in some contextualised form
Devisingaplan: findtheconnection between dataandtheunknown, consider auxiliary problemifanimmediateconnectioncannotbefound, andfinallyobtainaplanofthe solution	*Transformation:* transform a word problem into an appropriate mathematical problem	
Carryingoutthe plan	*Process skills*: perform mathematical procedures	*Employ:* apply mathematical concepts, facts, procedures, and reasoning to solve mathematically-formulated problems to obtain mathematical conclusions
Lookingback: examinethesolution obtained	*Encoding:* represent the mathematical solution into acceptable written form	*Interpret/evaluate:* reflect upon mathematical solutions, results, or conclusions and interpret them in the context of real-life problems.

Table 1. Comparison among Process of Solving Mathematical Task

Specifically, steps of understanding problem and devising strategies, simultaneously, has likely similar idea with steps of reading, comprehension, and transformation in Newman analysis, while this idea also appear on mathematical literacy, i.e. formulate. As early stages in solving mathematical task, they end up with determining precise mathematical model or strategies before performing further steps of solving problem. Likewise, each idea of carrying out step in Polya's process, process skill in Newman, and employ in PISA's mathematical literacy deals with undertaking mathematical procedure to find mathematical results, such as performing arithmetic computations, solving equations, making logical deductions from mathematical assumptions, performing symbolic manipulations, or extracting mathematical information from tables and graph. Furthermore, the last step of Polya's, i.e. looking back, corresponds to final stage of Newman analysis, i.e. encoding and PISA's mathematical literacy, i.e. interpretation. The idea of this stage is interpreting the mathematical result to the initial problem such as checking the reasonableness of the answer or considering other strategies and solution of the problem. Comparing those three frameworks, it is known that Polya's problem solving steps, which was introduced before the other two frameworks, have an agreement with both Newman analysis and PISA's mathematical literacy. Thus, modified Newman's error categories can be used to analyze teachers' level of performance in solving CbPST: comprehension error, transformation error, mathematical processing error, and interpretation error (see table 3).

METHOD

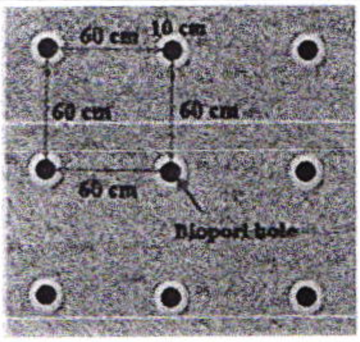
Participants were teachers from four cities in East Java province comprising 70 junior high school teachersand 55 senior high school teachers, have minimum a bachelor degree, andhave been taught more than five years. This selection method was applied since the need of having overview how experienced-teacher, particularly, emerge their content knowledge in their performance on solving CbPST.Data were obtained through context-based task and questionnaire. Teachers' work on the task was analyzed using framework on CbPST error (adapted from Newman analysis (Clements, 1980), OECD, 2013a, and Wijaya, et, 2014) while their response on questionnaire was analyzed using percentage.All the tasks were designed through a development process to achieve their validity and practicality, i.e. from designing initial draft, validating the draft to experts and learners, to examining the revised draft in a field testin order to obtain a final set of task which satisfied the framework of PISA task by OECD (2013a).We developed the task not only by considering the conformity of the task toward PISA framework but also by conjecturing sub type of responses which might be appeared on teachers' work as described in table 3.Therefore, when designing initial draft, we usedsome of the tasks from PISA like task developed by Kohar and Zulkardi (2015) which has been used to effectively analyze students' mathematical literacy based on this consideration, while others were own developed.The unit context for junior high school teacher are 'Futsal score' (1 item) and 'World Online Mathematics Contest' (1 item), while unit context for senior high school teacher is 'Biopori' (3 items). The task demand is given as follows.

Category of teacher	Unit of Context	No	Process Category	Description
Junior high school teacher	World online mathematics contest	1	Formulate	Formulate a mathematical model to determine a proper time to hold an online mathematics contest participated by students from different countries in different continents
	Futsal score	2	Interpret	Interpret information about a score achieved by a futsal team in a tournament which is implicitly stated from the information given in a table
Senior high school teacher	Biopori	3	Formulate	Formulate a mathematical model to determine the maximum number of biopori for a given field size
		4	Employ	Employ a set of mathematical procedures and facts to find the minimum number of biopori for a certain amount of waste

5	Interpret	Interpret the relationship between the diameter of a biopori hole and the maximum amount of waste that can be loaded by a bioporito sketch an appropriate graph representing that relationship

Table 2.CbPST cognitive demand

The number of tasks also consider the balance of major cognitive demand needed dominantly within the three mathematical processes: formulate, employ, interpret.Thus, there were at least two of those processes examined to each teacher category as shown in table 2. A further explanation of how we categorized item is given by someexamples. Task no 3, for instance, needs likely more cognitive demand when determining a precise mathematical model by connecting the relationship among information: the square pattern, distance between the hole, the field size. Hence, this item was categorized as 'formulate' item.



Zaki wants to make a number of biopori holes having a hole diameter of 10 cm in the lawn behind his house. He plans to make the arrangement of the biopori hole with a square pattern as shown in the figure.

Using this design, what is the maximum number of holes that can be made by Zaki if the size of the field is 10 m x 10 m? Explain your strategy

Another task category, 'interpret' item for instance, is shown by task 2 below.

To prepare a tournament, three futsal teams from Exacta school held a futsal training in their schoolyard once a week. Each of them face each other in a match exactly once. The following table presents the result of the training this week.

Team	Won	Lost	Drawn	Goals	
				Goals For	Goals Againts
Mentari	1	0	1	7	5
Surya FC	1	0	1	8	6
Rajawali	0	2	0	5	9

Note:
Won : The number of matches won
Lost : The number of matches lost
Drawn : The number of matches drawn
Goals for: The number of goals scored
Goals against: The number of goals conceded

What is the score of the match Mentari vs Rajawali? Explain your reason.

The questionnaire was developed to investigate teachers' most difficult process in performing context-based problem solving task. The question proposed was 'what is the most difficult step you experienced when solving problem solving task?'.A number of options offered to participants were (a) read the task to understand every single word or presented picture, (b) understand what is asking, (c) derive a precise mathematical model to solve the task, (d) employing mathematical procedures, such as algebraic operation ad equation, (e) interpret the mathematical result back to the initial problem.

RESULTS

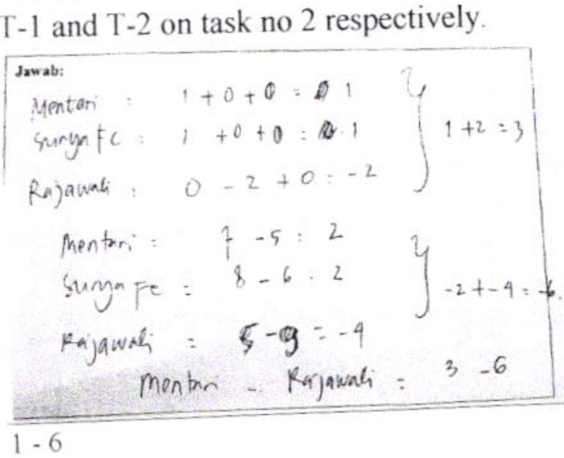
Teachers' ErrorinPerformingContext-basedProblemSolvingTask

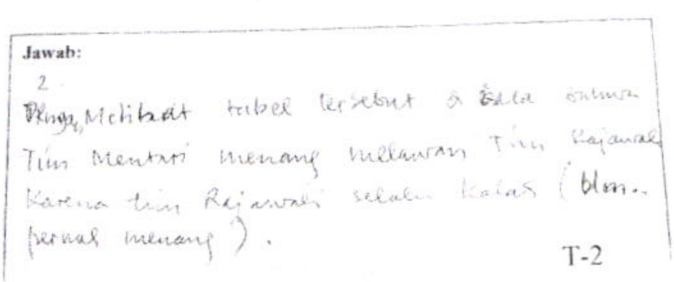
In total, we had 305 possible responses (number of tasks worked out by all teachers in total) which included 109 correct responses (35.73%), 109 incorrect responses (35.73%), i.e., no credit or partial credit, and 87 missing responses (28.54%). Each incorrect response had an opportunity to be coded by more than one sub-type code since its different errors found from this response.For instance, an incorrect response was possibly coded as T-1 and P-2 simultaneously. Thus, we had 498 coded responses, excluding both correct and missing responses.

Mathematical Process	Type of responses	Sub-type	Code	N	%
Formulate	Comprehension error	Misunderstanding the instruction	C-1	59	11,85%
		Error in selecting information	C-2	94	18,88%
	Transformation error	Procedural tendency	T-1	153	30,72%
		Taking too much account of the context	T-2	6	1,20%
		Wrong mathematical operation/concept	T-3	30	6,02%
		Treating a graph as a picture	T-4	8	1,61%
Employ	Mathematical Processing error	Algebraic error	P-1	6	1,20%
		Arithmetical error	P-2	15	3,01%
		Error in mathematical interpretation of graph	P-3	18	3,61%
		Measurement error	P-4	15	3,01%
		Unfinished answer	P-5	8	1,61%
Interpret	Interpretation error		I	86	17,27%

Table 3. Frequency of teacher's error on CbPST

An example of response showing error C-2 on task 3 is shown, for example, by teacher who divided the length of one side of the field size by the distance between two biopori hole (1000cm/60cm=16.66 or 17) and then calculate the number of biopori from 17x17=289 holes. Here, the teacher selected wrong information related to the number of biopori in one side of the field. Meanwhile, another error, T-3, was performed by teacher who used mathematical procedure/concepts which are not relevant to the tasks such as incorrect formula of n-series of arithmetical sequence or other algebraic form/equation which does not precisely represent the situation of the task.The biggest error frequency, T-1, was performed by teachers who tended to use directly a mathematical procedure (such as formula, algorithm) without analyzing whether or not it was needed, while the least number of error, T-2, was performed by teachers who only referred to the real world situation without taking the perspective of mathematics. The examples below show T-1 and T-2 on task no 2 respectively.


Jawab:
Mentari : 1 + 0 + 0 = 1
Surya FC : 1 + 0 + 0 = 1
Rajawali : 0 - 2 + 0 = -2
1 + 2 = 3
Mentari : 7 - 5 = 2
Surya FC : 8 - 6 = 2
Rajawali : 5 - 9 = -4
-2 + -4 = -6
Mentari - Rajawali = 3 - 6


Jawab:
2.
Dengan Melihat tabel tersebut di atas bahwa Tim Mentari menang melawan Tim Rajawali Karena tim Rajawali selalu kalah (blm.. pernah menang).
T-2

T-1

Translation:
2-1.By looking at the table, it is noted that Mentari won againts Rajawali because Rajawali always lost (never won)

Figure 1. Examples of errors: T-1, T-2

Teachers' Difficulties in Viewing Context-based Problem Solving Task

Teachers' view on mathematical processin a questionnaire resultshow that the process of determining precise mathematical model and strategies is admitted as most difficult step(70 out of 172chosen options) compared to the other three processes such as understanding problem (47 out of 172chosen options), employing mathematical procedures and facts (9 out of 172chosen options), and interpreting result back to initial problem (29 out of 172 chosen options). This results agreed with teachers' result on the task examined which point out that transformation error appear as the biggest number of error, i.e. 39,54%.

Teachers' Difficulties in Activating FMC within Mathematical Process

The following are the most common manifestations of these difficulties:

1. Communication: Difficulties in presenting an argumentation constructively in natural language rather than in formal/symbolic language. For example, the teachers did not show a strong argumentation presented for the task 2 in natural language which is typically used in their daily life. They seemed difficult to explain their strategy in finding the score of the match although they really know what the score is.
2. Reasoning and argumentation: Conveying incomplete inferences among the information.On task 2, teachers who failed to respond were not able to see the relationship between the information about 'win-lose', and goals for - goals againts in the table presented to find the score between Mentari and Rajawali. On task 1, there were many teachers who did not compared information about time zone of the cities in different countries with the allowable time for students to join the competition.
3. Mathematising: Using mathematical model which is not supported by a good understanding of related concepts. For example, some teachers calculated the maximum number of biopori holes on task 3 which could be built by firstly counting the area of the field, i.e. 1000cm x 1000cm = 1.000.000cm^2 then dividing this value by the area of a rectangular pattern, i.e. 80cm x 80cm = 6400cm^2. Thus, they got 156 holes. It shows that they had less understanding of the use of the concept of area in solving the task.
4. Representation: (1) Difficulties in determining a precise scheme to process information related to graph. On task 5, students were unable to make a mathematical relationship between diameter of circular base of a biopori hole and the maximum volume of waste that can be loaded by the hole. This difficulty can be seen as they did not catch with the idea of textual representation written in the body of the task. However, some teachers really knew about the trend of the graph should be, but when they were confronted with the idea of quadratic equation because of the existence of the circular base, they did not grasp about whether the graph will be linear or tend to be parabolic. (2) Difficulties in interpreting a model of figural representation to catch the situation. For example, on task 3, we found there were some teachers who failed in generalizing the square

pattern of biopori given in the figure into a larger pattern and These facts indicates that they failed in providing substantial description about what the figure means.

5. Using formal/symbolic language: Not precise in handling with standard unit of measurement in calculation. Some teachers were not carefully convert numerical information related to the field size and distance between two biopori in same unit of measurement (meter-centimeter) whenworking on task 3 and 4.

DISCUSSION AND CONCLUSION

From table 1 we can see that teachers had most errors in transformation (39,54%), i.e. providing mathematical structure to the problem presented in some contextualform.Regarding teacher's difficulties in a questionnaire result, teachers admitted to most frequently failed in determining a precise mathematical model or strategies when carrying out mathematical literacy processes: formulate.Being mathematically literate, one need to activate their FMC continuesly in all the mathematical processes: formulate, employ, and interpret. However, we also found that the teachers did not reach some indicators of FMC well shown by their difficulties in activating FMC whereas FMC, as Niss (2015) said, has important role to to underpin those process. This underpinning, for instance, emerges when activating communication as one of FMC. One need to activate receptive communication by making sense of every single word, procedure, questions or statements when formulating problem into a mathematical model, state the work of carrying out the mathematical model which is mathematically acceptable and obviously understood when employing mathematical procedures and fact, present and explain the work constructively when providing interpretation of the mathematical result (OECD, 2013a).

Our study seemed support a study of Siswono, Rosyidi, Kurniasari, and Astuti(2015) on Indonesian elementary teachers' problem solving which revealed that teachers had likely problem with their problem solving content knowledge such as finding strategies/mathematical model as early stages of solving mathematical problem.Problem solving content knowledge as Chapman (2015) said is important to understand what makes successful mathematical problem solving such as problem solving strategy, type of problems, as well as being proficient in solving those problemsusing problem solving steps though. Mathematical literacy content knowledge, on the other hand, has unique need of carrying out the three mathematical processes, and the mathematical competencies, i.e.FMC since the type of problems assessed which is real world authentic problemhaving unique characteristics as OECD (2013a) mentioned within its framework.The problem faced by teachers could then be further investigated by studying why teachers have weakness on performing CbPST and what learning opportunity they have had in learning problem solving as well as mathematical literacy from their source of teaching book, training teacher program, school learning environment, etc. Hence, the implication of this study suggest to provide a professional program to develop teacher's performance on context-based task as part of strengthening concept of being mathematically literate as well as applying idea of mathematical literacy in practical teaching.

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